The University of Hong Kong
Department of Electrical and Electronic Engineering

MSc. Experiment: Design of Linear Phase FIR Filters

1. Objectives:

The purpose of this experiement is to study the design of one dimensional finite duration impulse response (FIR) linear phase filters using MATLAB. The window method and the parks-McClellan method will be studied.

2. Equipment Required

PC-386/486,
PC-MATLAB 386.

References: A. V. Oppenheim : Discrete-time Signal Processing, Prentice-Hall, 1991.

3. Suggested Duration

One three hour laboratory session.

4. Theory

4.1. Window method

Window method generally begins with an ideal desired frequency response:

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j\omega n} \tag{1}$$

where $h_d(n)$ is the corresponding impulse response sequence. Many idealized systems are defined by piecewise-constant or piecewise-functional frequency responses with discontinuities at the boundaries between bands. As a result, they have impulse response that are noncausal and infinitely long. Window method truncates the infinitely long impulse response by multiplying it with a finite sequence called the window functions:

$$h(n) = h_d(n)w(n) \tag{2}$$

The Fourier transform of the FIR filter is then given by:

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})W(e^{j(\omega-\theta)})d\omega \tag{3}$$

which is the periodic convolution of the desired ideal frequency response with the Fourier tranform of the windows. Hence, $H(e^{j\omega})$ will be a smeared version of $H_d(e^{j\omega})$. The most popular window functions being the Kaiser window:

$$w(n) = \begin{cases} \dfrac{I_0[\beta(1-[(n-\alpha)/\alpha]^2)^{1/2}]}{I_0(\beta)}, 0 \le n \le M, \\ 0, otherwise \end{cases} \quad (4)$$

where $\alpha = M/2$, and I_0 represents the zeroth-order modified Bessel function of the first kind. The design procedure involves the determination of the length M and the shape parameter β from the given specification. Let

$\delta = \min\{\delta_1, \delta_2\}$

$\Delta\omega = \omega_s - \omega_p$ (for lowpass approximation)

where δ_1, δ_2 are respectively the pass and stop band ripples in the specification.

ω_s, ω_p are respectively the stop and pass band edge frequencies.

Define

$$A = -20\log_{10}\delta \quad (5)$$

Kaiser determined empirically that the valued of β and M needed to meet the specification is given by:

$$\beta = \begin{cases} 0.1102(A-8.7), A > 50, \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), 21 \le A \le 50, \\ 0.0, A < 21. \end{cases} \quad (6)$$

$$M = \frac{A-8}{2.285\Delta\omega} \quad (7)$$

Similar procedure can be applied to design multiband filters.

4.2. Parks-McClellan Method

The Parks-McClellan algorithm uses the Remez exchange algorithm and Chebyshev approximation theory to design filters with optimal ∞ error norm, i,e. the maximum error between the desired frequency response and the actual frequency response is minimized. Filters designed this way exhibit an equiripple behavior in their frequency response.

Unlike the Kaiser window method, the pass and stop band ripples need not be the same. Kaiser has obtained the following simplified formula for the estimation of filter order for a given specification:

$$M = \frac{-10\log_{10}(\delta_1\delta_2) - 13}{2.324\Delta\omega} \qquad (8)$$

4.3 PC-MATLAB

PC-MATLAB is an interactive environment for high level simulation of general numerical applications such as numerical linear algebra, signal processing, etc.

MATLAB works with essentially only one kind of object, a rectangular numerical matrix with possibly complex elements. Refer to PC-MATLAB Reference and Tutorial for more details of matrix declaration and manipulation.

User defined function or command files can be added to MATLAB's vocabulary if they are expressed in terms of other existing functions. The commands and functions that comprise the new function are put in a file whose name defines the name of the new function, with a file type of .m appended. See the PC-MATLAB reference pp.3-84-3.85 for details. M-files can be created using ordinary text-editer and placed in the same directory of MATLAB or other defined search path in MATLAB. Type the name of the M-file in the MATLAB environment will invoke the commands or functions defined in the corresponding M-file.

In this experiment, we shall make use of the two functions in the Signal Processing Toolbox the PC-MATLAB for designing linear phase FIR filters.

fir1 implements the classical method of windowed linear-phase FIR digital filter design. It is formulated to design filters in standard lowpass, bandpass, highpass and bandstop configurations. For example

b = fir1(n,Wn,kaiser(n+1,β)

returns row vector b containing the n+1 coefficients of the order n Kaiser windowed lowpass linear-phase FIR filter with normalized cutoff frequency Wn. Refer to MATLAB Signal Processing Toolbox Tutorial pp. 1-32 - 1.33 and 2-61 - 2-62 for more details.

remez is a function to design linear phase FIR filters using the Parks-McClellan algorithm.

b = remez(n,f,m,w)

returns row vector b containing the n+1 coefficients of the order n FIR filter whose frequency-magnitude characteristics match those given by vector f and m.

f is a vector of frequency points, specified in the range between 0 and 1, where 1.0 corresponds to half the sample frequency.

m is a vector containing the desired magnitude response at the points specified in f. The elements of m must appear in equal-valued pairs.

w is the weight used in each frequency band.

Refer to pp. 1-34 -1-35 and 2-91 - 2-93 of the Signal Processing Toolbox tutorial or the on-line help manual of the PC-MATLAB for more details.

5. Procedure

5.1 Kaiser Window Method

Low Pass Filter Design

Use the function **fir1** in MATLAB to design an FIR lowpass filter using Kaiser Window with the following specifications:

$$\omega_p = 0.4\pi,\ \omega_s = 0.6\pi,\ \delta_1 = 0.01,\ \text{and}\ \delta_1 = 0.001 \qquad (9)$$

Obtain the estimate of the filter from eqn. (5)-(7).

High Pass Filter Design

Design a highpass filter with the following specifications:

$$\omega_s = 0.35\pi,\ \omega_p = 0.5\pi,\ \delta_1 = 0.021,\ \text{and}\ \delta_1 = 0.021 \qquad (10)$$

Obtain an estimate of the filter length from eqn. (5)-(7).

b = fir1(m,ω_c,'high',Kaiser(m+1,β))

5.2 Parks-McClellan Method

Low Pass Filter Design

Use the function **remez** in MATLAB to design a lowpass filter with the same specification as in (9). Use formula (8) to estimate the filter order.

Band Pass Filter Design

$$H_d(e^{j\omega}) = \begin{cases} 0, 0 \le \omega \le 0.3\pi \\ 1, 0.35\pi \le \omega \le 0.6\pi \\ 0, 0.7\pi \le \omega \le \pi \end{cases} \qquad (11)$$

$$W(e^{j\omega}) = \begin{cases} 1, 0 \le \omega \le 0.3\pi \\ 1, 0.35\pi \le \omega \le 0.6\pi \\ 0.2, 0.7\pi \le \omega \le \pi \end{cases} \qquad (12)$$

Use a filter order of 75.

6. Discussion

3.1. Plot and discuss the frequency response of all the filters designed.
3.2. Comment on the two approaches in designing linear phase FIR filters.

SCC, March. 7, 1995.