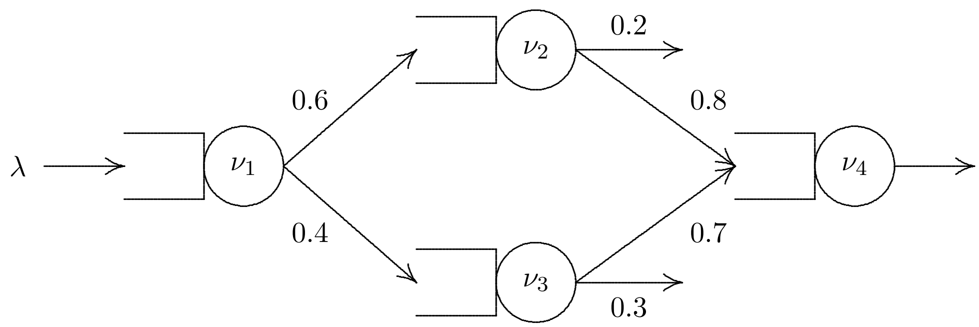
CS426/526 — Simulation — Fall 2008
Final Exam

This final exam is due at 8:30am on Tuesday December 16 and will not be accepted late without prior approval. For each question, you must provide: (*i*) a written discussion of the approached used, including equations and references to program modules as appropriate; (*ii*) a descriptive summary of all simulation outputs and conclusions; and (*iii*) program listings as appropriate. Be concise yet comprehensive. Verbose, rambling "shotgun" responses will be penalized. If you use any resource other than the text or your class notes, it must be properly cited.

You may not seek or receive assistance on this test from any person other than me. Failure to observe this rule will be treated as an honor code violation.

Problems 1 and 6 are each worth 50 points. Each of the other problems are worth 25 points. All students must work problems 1 and 6. Undergraduate students must work *any two* other problems (150 points total). Graduate students must work *all four* other problems (200 points total).

problem 1: Consider the network of single-server service nodes indicated. Jobs enter the network from the left at node 1 as a stationary Poisson process with rate $\lambda = 4.0$. As a job departs node 1, it goes either to node 2 with probability 0.6 or to node 3 with probability 0.4. As a job departs node 2 it either leaves the network (because service is completed) with probability 0.2 or it goes to node 4 with probability 0.8. Similarly, as a job departs node 3 it either leaves the network with probability 0.3 or goes to node 4 with probability 0.7. As a job departs node 4 it leaves the network.



The service discipline at each service node is FIFO, the queue capacity is infinite, and the service rate is ν. The service time characteristics are

node 1	*Erlang* (4, 0.04)	($\nu_1 = 1/0.16$)
node 2	*Erlang* (6, 0.05)	($\nu_2 = 1/0.30$)
node 3	*Erlang* (5, 0.10)	($\nu_3 = 1/0.50$)
node 4	*Erlang* (3, 0.08)	($\nu_4 = 1/0.24$)

(*i*) Based on *many* jobs processed by the network, estimate the steady-state utilization of each service node. (*ii*) Similarly, estimate the time-averaged number of jobs in the queue at each service node. (*iii*) What is the expected time spent in the network (per job)? (*iv*) What did you do to convince yourself that your results are correct?

problem 2: Based on your knowledge of simulation, you are hired as a consultant to help build a computer-based simulation of the card game *Idiot's Delight*. (The game will be played in locations where gambling is legal.)

A conventional 52-card deck is used – four suits, 13 cards per suit. Cards are drawn, one at a time, from a well-shuffled *deck* and placed into a *hand*. The cards in the hand are maintained in the order they were drawn from the deck. The *top* card in the hand, is the one that is most recently drawn. The *reference* card in the hand is the card three positions below the top card. The *between* cards in the hand are the two cards between the top card and the reference card.

Each time a card is drawn, it becomes the top card and, if there are 4 or more cards in the hand, the reference card is defined accordingly. The top card is the compared with the reference card and

(a) if the two cards agree in rank, then the top card, the two between cards, and the reference card are discarded;

(b) else if the two cards agree in suit, then the two between cards are discarded;

(c) else no cards are discarded.

When cases (a) or (b) occur, if four or more cards remain after the discard, then a new reference card is defined accordingly and compared with the top card to see if additional discards are possible.

The game proceeds in this way until all cards have been drawn from the deck and no more discards from the hand are possible. No discards are possible unless there are at least 4 cards in the hand. If there are less than 4 cards in the hand then, if possible, additional cards are drawn from the deck until there are 4 cards in the hand.

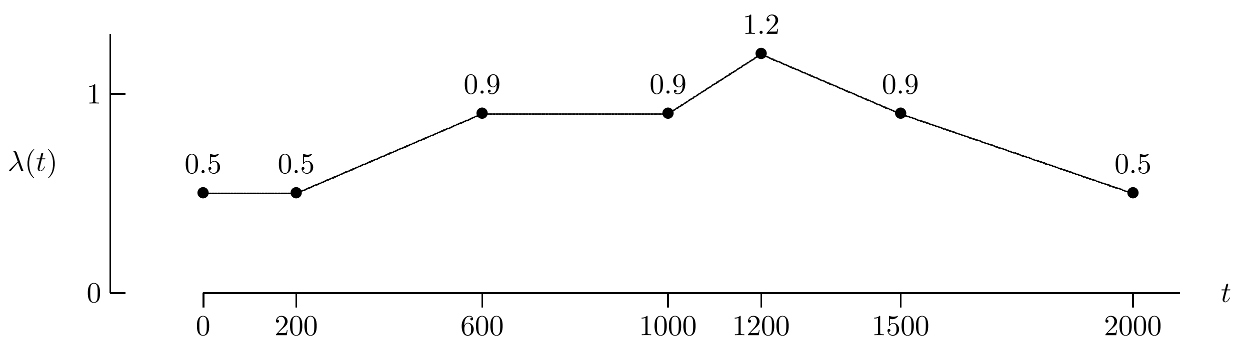
Players will be charged D dollars per play of of the game, with the understanding that they will win \$1 for each card they discard during the play of the game, unless they discard *all* 52 cards in which case they will win \$10 000. Your job is to determine D; that is, determine the smallest value of D (an integer) so that the expected owner profit per play of the game is at least \$5. (That is, the expected player loss is at least \$5 per play of the game.) (*i*) What is the value of D? (*ii*) As a consultant, you are expected to justify your answer (and your fee) — do so. (*iii*) As a scientist you feel compelled to convey some sense of the uncertainty that is inevitably associated with decisions based upon the modeling and analysis of a stochastic system — do so. (*iv*) What did you do to convince yourself that your results are correct?

problem 3: Simulate a single-server service node for which the server uses a shortest-job-first priority queue discipline based upon a knowledge of the service time for each job in the queue. Assume a stationary Poisson arrival process with rate $\lambda = 10.0$ and assume that the service times are *Erlang*(4, 0.02) random variables. (i) Estimate the steady-state value of $\bar{l}$, $\bar{q}$, and $\bar{x}$. (*ii*) Compare these estimates with the corresponding steady-state estimates if the queue discipline is FIFO. (*iii*) What did you do to convince yourself that your results are correct? (*iv*) Comment on the value of this priority queue discipline.

problem 4: Suppose you play the "win in any order" version of Pick-3 (see exercise 6.1.7) each day for 365 days. You start with \$365 and it costs you \$1 each day to play. Let the discrete random variable X represent the amount of money you will have after 365 days, including the \$80 wins, if any. The possible values of X are 0, 80, 160, 240, ... Use Monte Carlo simulation to estimate (*i*) the pdf of X; (*ii*) the expected amount of money you will have (as a 95% confidence interval) after 365 days; (*iii*) the probability (as a 95% confidence interval) that you are a "winner" (i.e., have more than \$365 dollars) after 365 days. As an alternative to Monte Carlo simulation, use the appropriate functions in the library `rvms` to compute the theoretical values corresponding to what you estimated in parts (*i*), (*ii*), and (*iii*). (*iv*) Comment on the value of playing this game as a long-term investment strategy.

problem 5: Two integers X_1, X_2 are drawn at random, without replacement, from the set $\{1, 2, \ldots, n\}$ with $n \geq 2$. Let $X = |X_1 - X_2|$. (*i*) What are the possible values of X? (*ii*) What are the pdf, cdf, and idf of X? (*iii*) What are the mean and standard deviation of X? (*iv*) Construct an algorithm that will generate a possible value of X with just one call to `Random()`. (*v*) Present convincing *numerical* evidence that your algorithm is correct for the case $n = 11$.

problem 6: Use algorithm 7.5.4 with $\tau = 2000$ to construct a finite-horizon simulation of an initially idle single-server service node with a nonstationary Poisson arrival process. Assume that the arrival rate $\lambda(t)$ is the piecewise-linear spline illustrated and that the service time distribution is *Erlang* (4, 0.25).



(*i*) The simulation should be replicated 64 times to construct a 95% confidence interval estimate for the mean of the instantaneous (snapshot) number in the node at each of the $\lambda(t)$ time knots (other than $t_0 = 0$). (*ii*) Comment on how these interval estimates relate to the nonstationary nature of the arrival process. (*iii*) If you were to approximate the nonstationary Poisson arrival process with an "equivalent" stationary arrival process with constant rate $\bar{\lambda}$, what would be the numerical value of $\bar{\lambda}$?

Start early and create something of which you, and I, can be proud.

– E. Smirni, 11-25-2008